A Survey of South African Grade 10 Learners’ Geometric Thinking Levels in Terms of the Van Hiele Theory

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ABSTRACT The main thesis of the van Hiele theory is that childrens’ understanding of geometric concepts can be classified into a sequence of five hierarchical thinking levels, with levels 1 and 5 being the lowest and the highest. However, an additional lower level, level 0, was added in by other researchers. This paper reports on a part of a larger study which focused on the van Hiele levels of geometric thinking amongst a group of Grade 10 learners. The sample consisted of 191 Grade 10 learners from five senior secondary schools in one Education District in the Eastern Cape Province of South Africa. The respective mathematics teachers in each of these schools assisted in the selection process. The schools were selected through purposive sampling. The necessary ethical requirements were met. Participants completed a test on van Hiele levels of geometric thought by responding to questions on basic geometric concepts including the classification and properties of triangles and quadrilaterals which constituted the basis for space and shape component of the South African Grade 10 Mathematics curriculum. The data were analyzed by manual counts and by using Microsoft Excel. The study found that the majority of learners were at level 0, which is a cause for concern. The paper recommends that educators who facilitate geometry learning in grade 10 need to familiarize themselves with the van Hiele levels in order to achieve effectiveness in the teaching/learning interface of geometrical concepts.

INTRODUCTION

Internationally, concern with difficulties in learning geometry is not new and can be traced back to several decades (for example, Usiskin 1982; Fuys et al. 1988; Gutierrez et al. 1991; Clements and Battista 1992). Findings from these studies indicate that many learners in both middle and high schools encounter difficulties and show poor performance in geometry. In South Africa too similar results have been reported (for example, De Villiers and Njisane 1987; King 2003; Atebe 2008).

This research project to a large extent was inspired by Fuys et al.’s (1988) interpretation of the van Hiele theory, Atebe’s (2008) study on van Hiele model of thinking and conception in plane geometry and the researchers’ experiences and concerns regarding the poor geometry performance of learners in many South African high schools. It sought to find the level of geometric thinking of the learners by using the van Hiele theory together with the results of subsequent research as a framework in determining the van Hiele levels of grade 10 learners in some selected senior secondary schools.

Geometry consists of a complex network of interconnected concepts which demand representational systems and reasoning skills in order to conceptualize and analyze not only physical but also imagined spatial environments. The National Council of Teachers in Mathematics (NCTM) entitled ‘Standards 2000’ document suggests that instructional programmes in mathematics should pay attention to geometry and spatial sense so that, amongst other things, learners use visualisation and spatial reasoning to solve problems both within and outside of mathematics (NCTM 2000). Geometric reasoning consists of the invention and use of formal conceptual systems to investigate shape and space (Battista 2007). Geometry focuses on the development and application of spacial concepts through which children learn to represent and make sense of the world (Thompson 2003). The Conference Board of the Mathematical Sciences (CBMS) observes that learning of geometry is usually confronted by conceptual difficulties (CBMS 2001). Teaching and learning of
geometry still remain as one of the most disappointing experiences in many schools across nations (Atebe and Schafer 2009).

Clements and Battista (1992) cite studies by Psychkalo (1968) and Wirszup (1976) which concluded that the difficulties in geometry impelled a lot of research by educators in the Soviet Union from 1930-1950. The aim of those studies was to find the source of this problem. These initial efforts by the Soviets brought only little progress (Pusey 2003). A variety of models to describe children’s spatial sense and thinking have been proposed and researched and these include Piaget and Inhelder’s Topological Prima

cy Thesis, van Hiele’s Levels of Geometric Thinking and Cognitive Science model (Clements and Battista 1992). However, the theoretical frameworks on geometrical thinking proposed by Piaget and that of the van Hieles tended to have attracted more attention than many others in terms of impacting on geometry classroom instructional practices. Thereafter, a lot of research was done to question and validate the van Hiele theory (Burger and Shaughnessy 1986; Fuys et al. 1988; Gutierrez et al. 1991; Wu and Ma 2006). Although the theory was primarily aimed at improving teachers’ as well as learners’ understanding of geometrical concepts, it also appealed as an ideal model for use as a theoretical framework as well as a frame of reference to link geometry to educational principles (King 2003). Among other researchers, King (2003), and Atebe (2008) observed that the van Hiele theory can be used to explain the geometric thinking of school learners. The main thesis of the van Hiele theory is that children’s understanding of geometric concepts can be characterized as being at a certain level within a range of hierarchical levels (Mayberry 1983).

The van Hiele Levels and Their Characteristics

According to the van Hiele theory, there are 5 levels of thinking that schoolchildren pass through in their acquisition of geometric understanding (Pegg and Davey 1998; Malloy 2002):

Level 1: Recognition (or Visualization): Learners at this level recognize a geometric shape by its appearance alone. Learners can identify, name and compare geometric shapes such as triangles, squares and rectangles in their visible form (Fuys et al. 1988). Properties of a figure play no explicit role in the identification process (Pegg and Davey 1998).

Level 2: Analysis (or Descriptive Level): Learners at this level identify a figure by its properties, which are seen as independent of one another (Pegg and Davey 1998). Learners analyze the attributes of shapes, some relationships among the attributes and discover properties and rules through observation (Malloy 2002). Learners can recognize and name properties of geometric figures, but they do not yet understand the difference between these properties and between different figures (van Hiele 1986).

Level 3: Informal Deduction (or Order): Learners at this level discover and formulate generalizations about previously learned properties and rules and develop informal arguments to justify those generalizations (Malloy 2002). They no longer perceive figures as consisting of a collection of discrete, unrelated properties. Rather, they now recognize that one property of a shape proceeds from another. They also understand relationship between different figures (Pegg and Davey 1998). Class inclusions are understood at this level (van Hiele 1999).

Level 4: Deduction: Learners at this level prove theorems deductively and understand the structure of the geometric system (Malloy 2002). They understand necessary and sufficient conditions and can develop proofs rather than relying on rote
learning. They can construct their own definitions of shapes (Pegg and Davey 1998).

**Level 5: Rigor.** Learners at this level can establish theorems in different systems of postulates and can compare and analyze deductive systems (Fuys et al. 1988; Malloy 2002).

As a consequence of some learners not achieving even the basic level (level 1), researchers have suggested the introduction of another level, called level 0. Clements and Battista (1990) named this level 0 as ‘pre-recognition’. They defined it by stating that “…children initially perceive geometric shapes, but attend to only a subset of a shape’s visual characteristic. They are unable to identify many common shapes” (Clements and Battista 1990: 354). In this study, the possibilities of the existence of level 0 were also considered in assigning the van Hiele levels.

Van de Walle (2004) suggests that in addition to the key concepts of the theory, there are four related characteristics of the levels: (i) the levels are sequential, that is, for a learner to operate successfully at a particular level, that learner must have acquired the strategies and knowledge of the preceding levels; (ii) the levels are not age-dependent, that is, progress from one van Hiele level to the next higher one is dependent more on an instructional experience than on biological maturation; (iii) geometric experience is the greatest single factor influencing advancement through the levels: the nature and quality of the experience in the teaching and learning program has a major impact on the advancement through the levels; (iv) when instruction is at a level higher than that of the learner, there will be an inadequate or even lack of effective communication between the educator and learner, which disadvantages the learner.

Nevertheless, Pegg and Davey (1998) observe that even though the descriptions are content specific, van Hiele’s levels are actually stages of cognitive development although as cited earlier, there are claims that the progression from one level to the next is not always the result of natural development or maturation, although these factors also may play a role. According to Malloy (2002), in ideal circumstances, learners from pre-kindergarten through high school are meant to think and reason about geometry in a similar progression as follows: from pre-kindergarten to grade 2 on the visualization level; grades 2-5 on the analysis level, grades 5-8 on the informal deduction level and grades 8-12 on the deduction level. But Malloy herself observes that usually, this is not the case. The quality and nature of the experience in the teaching and learning program ought to influence the advancement from a lower to a higher level.

This study sought answer to the following question: What are the van Hiele levels of geometrical thinking amongst the grade 10 learners?

**METHODOLOGY**

This was a quantitative design which made use of a multiple-choice test. The research was conducted at different sites. The sample consisted of 191 Grade 10 learners drawn up from five senior secondary schools in one District. These schools were selected through purposive sampling. Geographical accessibility, proximity and functionality were some of the factors that influenced the choice of these schools. These were semi-urban schools and drew learners from lower to middle income socio-economic communities. The schools were labeled by using alphabets A-E in order to ensure ethical conformity to safeguard their anonymity.

Formal approvals from the Department of Education and all school Principals were obtained in order to conduct this research. A research information sheet and an ‘informed consent’ form were given to all members of the sample or the parents in the case of learners below 18 years. The learners or parents of those below 18 years signed the ‘informed consent’ form. Anonymity of the schools and the learners was assured.

The research instrument (van Hiele geometry test) was a test question paper together with a multiple choice answer sheet based on the van Hiele levels of geometric thinking. The content was drawn from topics such as basic geometric concepts and classification and properties of triangles and quadrilaterals. These topics form the basis for space and shape in Grade 10 and upwards. Provision was made to assess the level of thinking across different concepts. The words items and questions are used interchangeably in this paper.

McMillan and Schumacher (2006) explain that the advantage of using standardized tests is that they are prepared by experts and in these tests...
careful attention has been paid to the nature of the
norms, reliability and validity because they are
intended to be used in a wide variety of settings.
The van Hiele model was developed in the 1950s
by Pierre van Hiele and Dina van Hiele Geldof.
Following that, Usiskin (1982) developed the van
Hiele Geometry Test which is known as Cognitive
Development and Achievement in Secondary
School Geometry (CDASSG) to test the theory and
since then, both the test and theory got refined
and thousands of people were tested with it. Atebe
(2008) adapted the above test and validated it by
consulting the geometry curriculum and the grade
10 mathematics text books. He also consulted two
experts, one in geometry and one in geometry
education for cross checking before making use of
his instrument in pilot testing for his study and
refined the instruments based on the feedback from
the piloting. Atebe (2008) carried out a recent
doctoral study which included learners in the
Grahamstown area within the same province where
this study was also carried out. The present
researchers adopted the Atebe (2008) instrument
with his permission.
There were 20 items in the test: items numbered
1-5, 6-10, 11-15 and 16-20 for identifying van Hiele
levels 1, 2, 3 and 4, respectively. Level 5 items were
not included since these were not expected at grade
10 level. The item numbers in the instrument are
repeated in the numbering of the sample items (see
Appendix A)
The mathematics educators from different
schools administered the instrument during school
hours in the respective classrooms as per the
instructions given by the researchers. Each member
of the sample was given a copy of the instrument.
They were requested to mark the letter option that
best represented their choice out of the five given
options. All the members of the sample in one school
completed the answer sheet at the same session. In
a report-back after the administration, the educators
reported that the learners appeared to follow the
instructions while they answered the questions
since no one asked for clarifications.

DATA ANALYSIS, RESULTS AND
DISCUSSION
The test scripts were scored by the researchers
using Microsoft Excel spreadsheet and these entries
were later verified by a grade 10 mathematics
educator.
Assignment of Learners to Levels
The learners were assigned to the levels using
a grading method which was based on the ‘3 of 5
correct’ success-criterion as suggested by Usiskin
(1982) and used by Atebe (2008). By this criterion,
if a learner answers correctly, at least 3 out of the 5
items in any of the 4 subtests within the test, the
learner was considered to have mastered that level.
According to this grading method, the learners’
scores were weighted as follows: 1 point for meeting
criterion on items 1-5 (level 1); 2 points for meeting
criterion on items 6-10 (level 2); 4 points for meeting
criterion on items 11 - 15 (level 3); 8 points for
meeting criterion on items 16-20 (level 4).

On application of the above, the maximum score
for any learner will be $1+2+4+8 = 15$ points. This
weighted sum helped to determine the van Hiele
levels on which the criterion has been met from the
weighted sum scores alone. The weighted sum and
the corresponding levels are shown in Table 1.

Table 1: van Hiele levels and weighted sums

<table>
<thead>
<tr>
<th>van Hiele levels</th>
<th>Corresponding weighted sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 2: van Hiele level of geometric thinking of learners in the five schools and all schools

<table>
<thead>
<tr>
<th>van Hiele level</th>
<th>School A (N and %)</th>
<th>School B (N and %)</th>
<th>School C (N and %)</th>
<th>School D (N and %)</th>
<th>School E (N and %)</th>
<th>Total (N and %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>23 (70%)</td>
<td>14 (32%)</td>
<td>3 (11%)</td>
<td>26 (70%)</td>
<td>25 (50%)</td>
<td>91 (48%)</td>
</tr>
<tr>
<td>Level 1</td>
<td>3 (9%)</td>
<td>20 (45%)</td>
<td>14 (52%)</td>
<td>6 (16%)</td>
<td>12 (24%)</td>
<td>55 (29%)</td>
</tr>
<tr>
<td>Level 2</td>
<td>4 (12%)</td>
<td>6 (14%)</td>
<td>6 (22%)</td>
<td>5 (14%)</td>
<td>6 (12%)</td>
<td>27 (14%)</td>
</tr>
<tr>
<td>Level 3</td>
<td>3 (9%)</td>
<td>4 (9%)</td>
<td>4 (15%)</td>
<td>0 (0%)</td>
<td>7 (14%)</td>
<td>18 (9%)</td>
</tr>
<tr>
<td>Level 4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>33 (100%)</td>
<td>44 (100%)</td>
<td>27 (100%)</td>
<td>37 (100%)</td>
<td>50 (100%)</td>
<td>191 (100%)</td>
</tr>
</tbody>
</table>
A score of 7 indicated that the learner met the criteria at levels 1, 2 and 3 (that is, $1+2+4 = 7$). This grading system helped to assign the learners into various van Hiele levels based on their responses. A weighted sum score of 0 indicated that the learner did not achieve any of the levels, that is, the learner did not get at least 3 out of 5 in any subtest of the test. Such a learner was thus operating at the pre-recognition level (that is, van Hiele level 0).

### Categorising Learners in Each van Hiele Level

Table 2 shows the number and percentage of learners on each of the van Hiele levels for the participating schools.

Figure 1 depicts the spread of the levels in terms of percentages of learners per school and for the entire study sample.

As shown in Table 2, the level-wise analysis for all schools for different levels were: level 0 at 48%; level 1 at 29%; level 2 at 14%; level 3 at 9% and level 4 at 0%. As can be seen in Table 2 and Figure 1, many learners were at level 0 with Schools A and D at 70%, School E at 50%, School B at 32% and School C at 11%. The low percentages of learners in Level 3 in all schools, in descending order were School C (15%), School E (14%), Schools A and B (9%), and School D (0%) and this should be a matter of concern. School C had learners with more conceptual base than all other schools with 52% at level 1, 22% at level 2 and 15% at level 3 (that is, 89% in levels 1-3) followed by School B with 68% of the learners on levels 1-3). The data show that learners in Grade 10 do not understand the relationship between different figures. None of the schools had learners that attained level 4 thinking on the van Hiele scale, indicating that the learners are not ready for formal geometric proofs in grade 10.

The assignment of learners into levels showed the percentage of learners in level 0 was almost 50% (48% or 91 out of 191 learners). This is much worse than those in the results from Usiskin (1982) who found that only 9% (222 out of the 2361 learners) were at level 0 and worse than that from Atebe (2008) which indicated 41% (29 out of 71 learners) at level 0. The Atebe study as well as this study indicate the difficulty that the South African learners have in recognizing figures in non-standard positions. Only 29% of learners in the sample could identify geometrical figures and only 14% were capable of identifying a figure by its properties. The low achievement in level 3 (9% of the sample) and 0% in level 4 showed that the learners were not ready for formal proof in Euclidean Geometry which represents the levels expected of senior secondary school learners. This calls for the need to deliver instruction at a level appropriate to learners’ level of thinking on the one hand and improving the quality of instruction starting from lower levels on the other hand.

### CONCLUSION

Educators are constantly concerned with the poor performance of learners in geometry. The van Hiele theory was useful in analyzing the performance of the learners. The results of this research pointed to some factors that could explain why learners experience difficulties with school geometry in the schools in which this research was carried out. A possibility in this regard is that some of the learners in the study had limited exposure to geometric figures. This shows the negative effect of not having prior learning on the successful movement through the levels. This research supports Clements and Battista (1990) on the existence of a level called prerecognition. van Hiele...
(1986) suggests that a group of learners having started homogeneously do not pass the next levels of thinking at the same time. This research also supports the unavoidability of such a situation. According to van Hiele (1986), the levels of thinking have a hierarchical arrangement, in the same sense that a learner cannot operate with understanding on one level without having been through the previous levels. Mayberry (1983), Pegg and Davey (1998) and the present study also support the hierarchical nature of the thinking levels, although the studies were carried out in different continents. The results from this study confirm previous insights into learning/teaching interface that take place in the geometry classrooms in some South African schools also.

**RECOMMENDATIONS**

The spatial orientation of learners should be developed and enhanced through the use of teaching aids and manipulatives in the class room. Learners must understand that geometric shapes are defined by their properties and not by their orientations in space. Educators need to provide learners with activities for discovering the properties of simple geometric shapes in different orientations. Success in learner attainment depends on the delivery of instruction that is appropriate to learners' level of thinking. With suitable instructional guidance from the educators, learners can formulate their own definitions of various shapes. Van Hiele (1986) postulated that learners' difficulty with school mathematics generally and geometry in particular is caused largely by educators' failure to deliver instruction that is appropriate to the learners' geometric level of thinking. Thus, the progress from one level to the next depends on the quality of instruction. Curriculum developers and text-book writers need to look into van Hiele theory for clues on how to improve learner achievement in mathematics in general and geometry in particular.

**ACKNOWLEDGEMENTS**

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**REFERENCES**


**APPENDIX A**

2.3.2.1. Level 1:

Question 1: Which of these are triangles?

![Sample item for level 1](image1.png)

A. All are triangles
B. 4 only
C. 1 and 2 only
D. 3 only
E. 1 and 4 only

2.3.2.2. Level 2:

Question 10: RSTU is a square. Which of these properties is not true in all squares?

![Sample item for level 2](image2.png)

A. RS and SU have the same measure.
B. The diagonals bisect the angles.
C. RT and SU have the same measure.
D. RT and Su are lines of symmetry.
E. The diagonals intersect at right angles.

2.3.2.3. Level 3:

Question 12: Which of the following is true?

A. All properties of rectangles are properties of all parallelograms.
B. All properties of squares are properties of all rectangles.
C. All properties of squares are properties of all parallelograms.
D. All properties of rectangles are properties of all squares.
E. None of (A) – (D) is true

2.3.2.4. Level 4:

Question 17: Examine the following statements.

i) Two lines perpendicular to the same line are parallel.
ii) A line perpendicular to one of two parallel lines is perpendicular to the other.
iii) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines S and P are perpendicular and lines T and P are perpendicular.

![Sample item for level 4](image3.png)

Which of the above statements could be the reason that line S is parallel to line T?

A. (i) only
B. (ii) only
C. (iii) only
D. Either (ii) or (iii)
E. Either (i) or (ii)